Phys 371
Fall 2020
Prof. Steven Anlage

Lecture 10, Blackbody Radiation

Monday 5 October, 2020

Front row: Michelson, Einstein, Millikan at the Athenaeum at Caltech in 1931

Blackbody Radiation

All objects at finite temperature *T* emit electromagnetic radiation

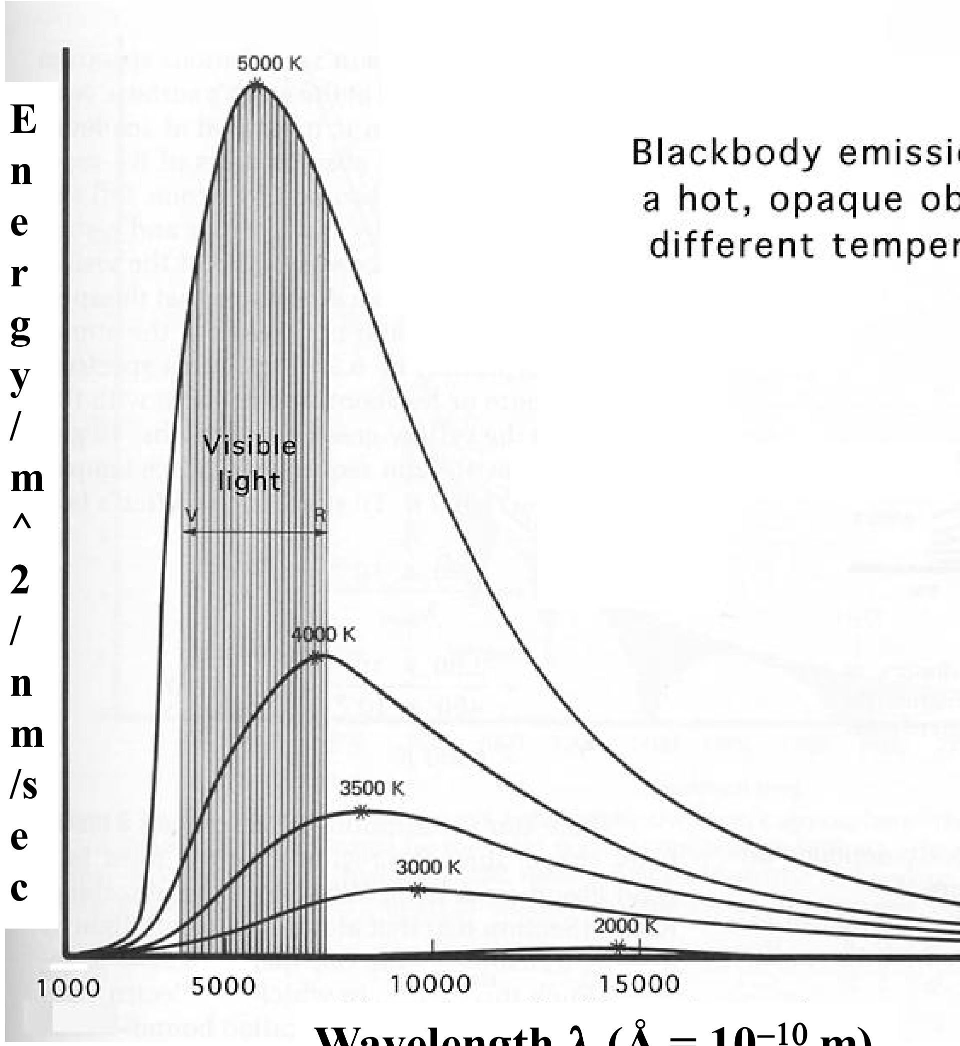
The total amount of radiation increases with the temperature of the object

$$\boldsymbol{R_{Total} = \sigma T^4}$$

$\boldsymbol{R_{Total}}$	**Total radiated power per unit area of object (W/m^2)**
$\boldsymbol{T}$	**Absolute temperature (K)**
$\boldsymbol{\sigma}$	**Stefan constant** $\boldsymbol{\sigma = 5.6703 \times 10^{-8}\, W/(m^2 K^4)}$

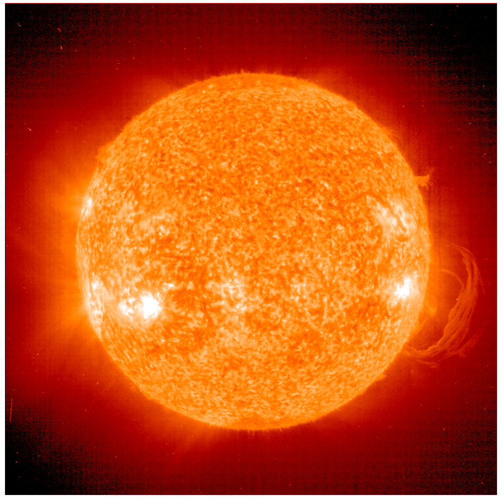
Blackbody Radiation is Widely Distributed in Wavelength / Frequency

$R(\lambda)$ (W/(m^2-nm))



Wavelength λ (Å = 10^{-10} m)

Blackbody emission from a hot, opaque object at different temperatures



$$R_{Total} = \int_0^\infty R(\lambda)\, d\lambda$$

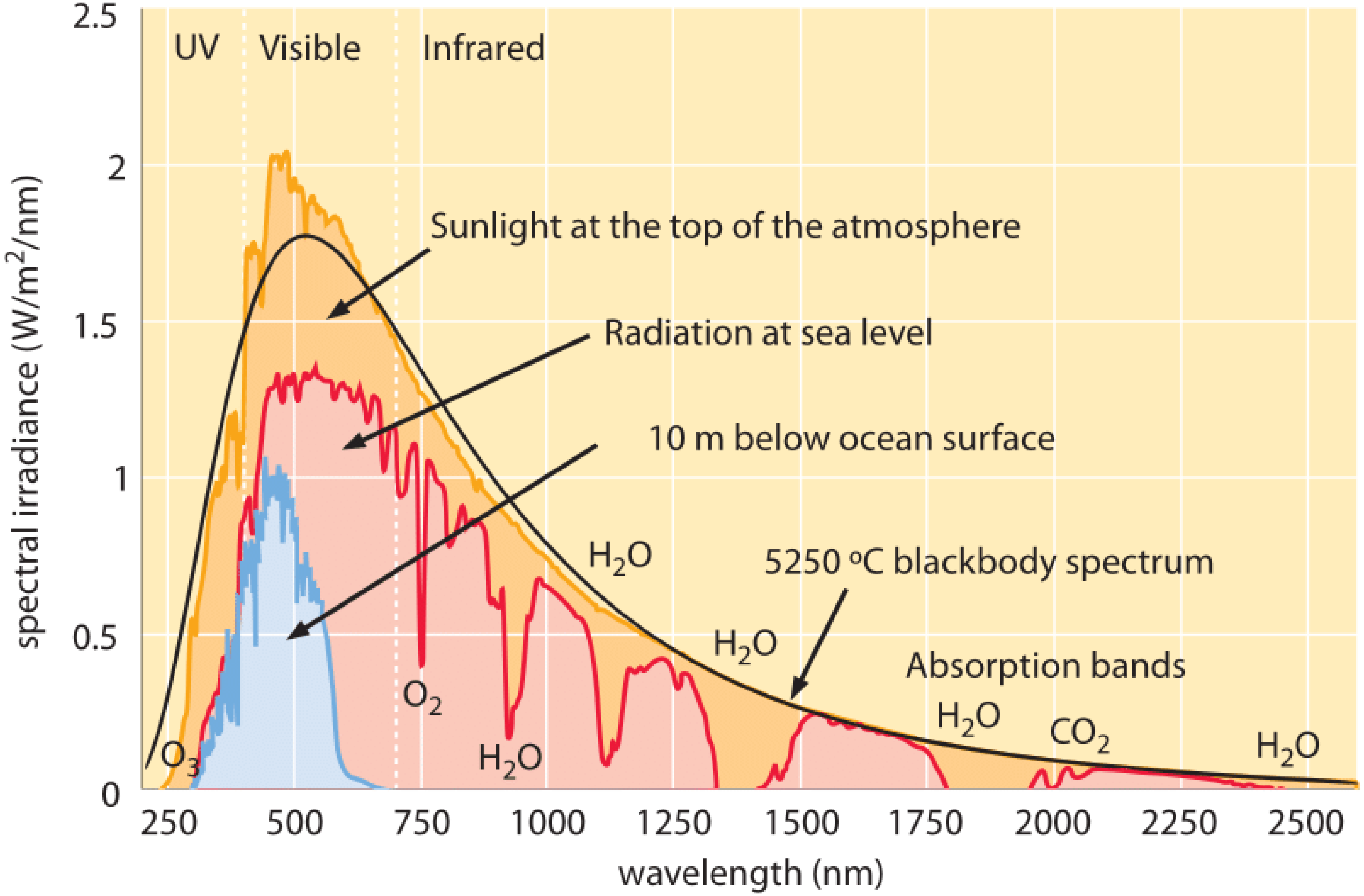
Blackbody Radiation

Thomas Wedgwood 1792

objects in kilns all become red at same temperature
independent of composition

This radiation spectrum is universal in the sense that all objects show the same emission spectrum as long as they are the same temperature. ***Universality*** **begs explanation …**

Solar Blackbody Spectrum
Measured Above and Below the Atmosphere



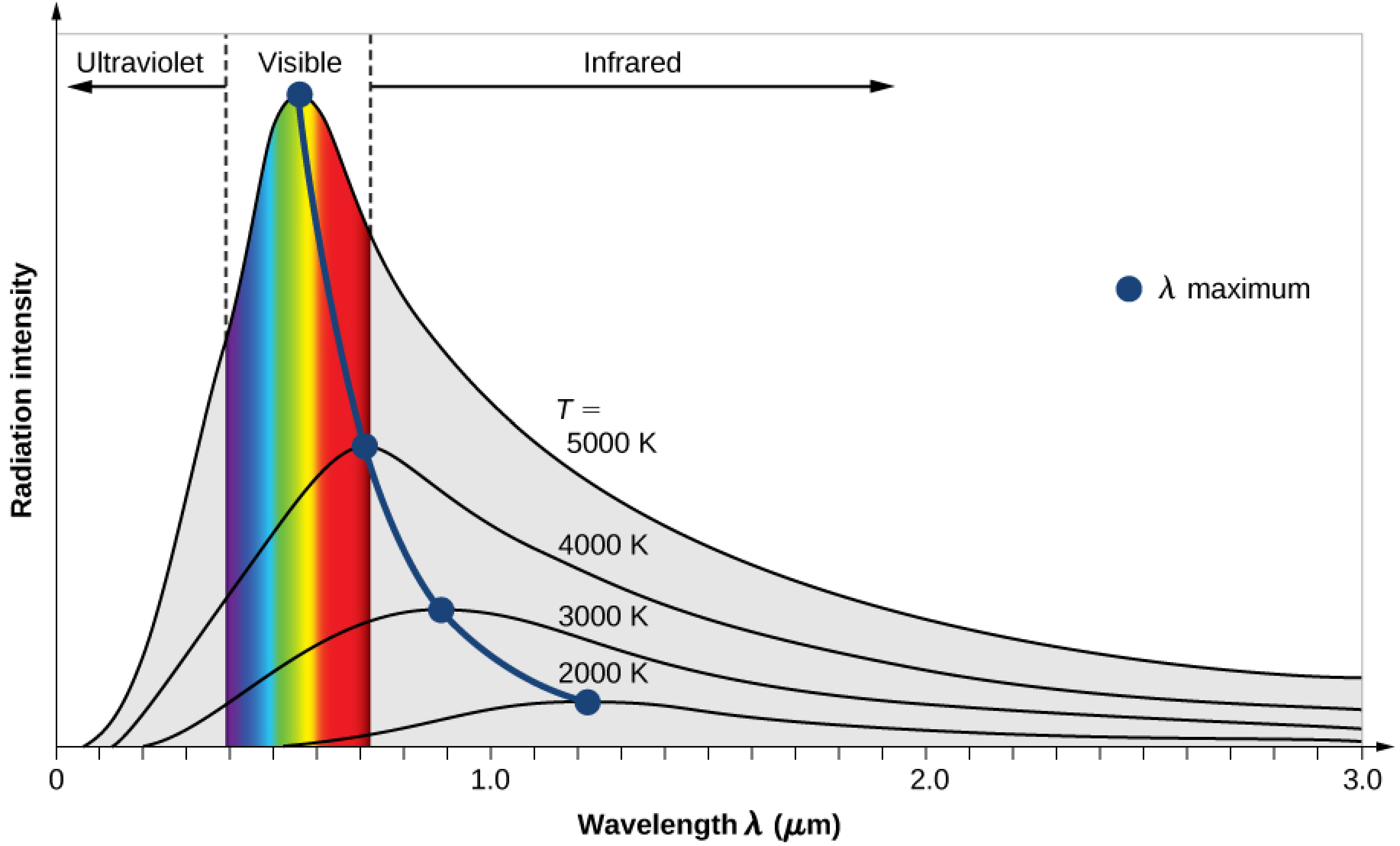
http://book.bionumbers.org/how-much-energy-is-carried-by-photons-used-in-photosynthesis/

The discrete absorption "lines" suggest that atoms have quantized energy levels, but that will come up later …

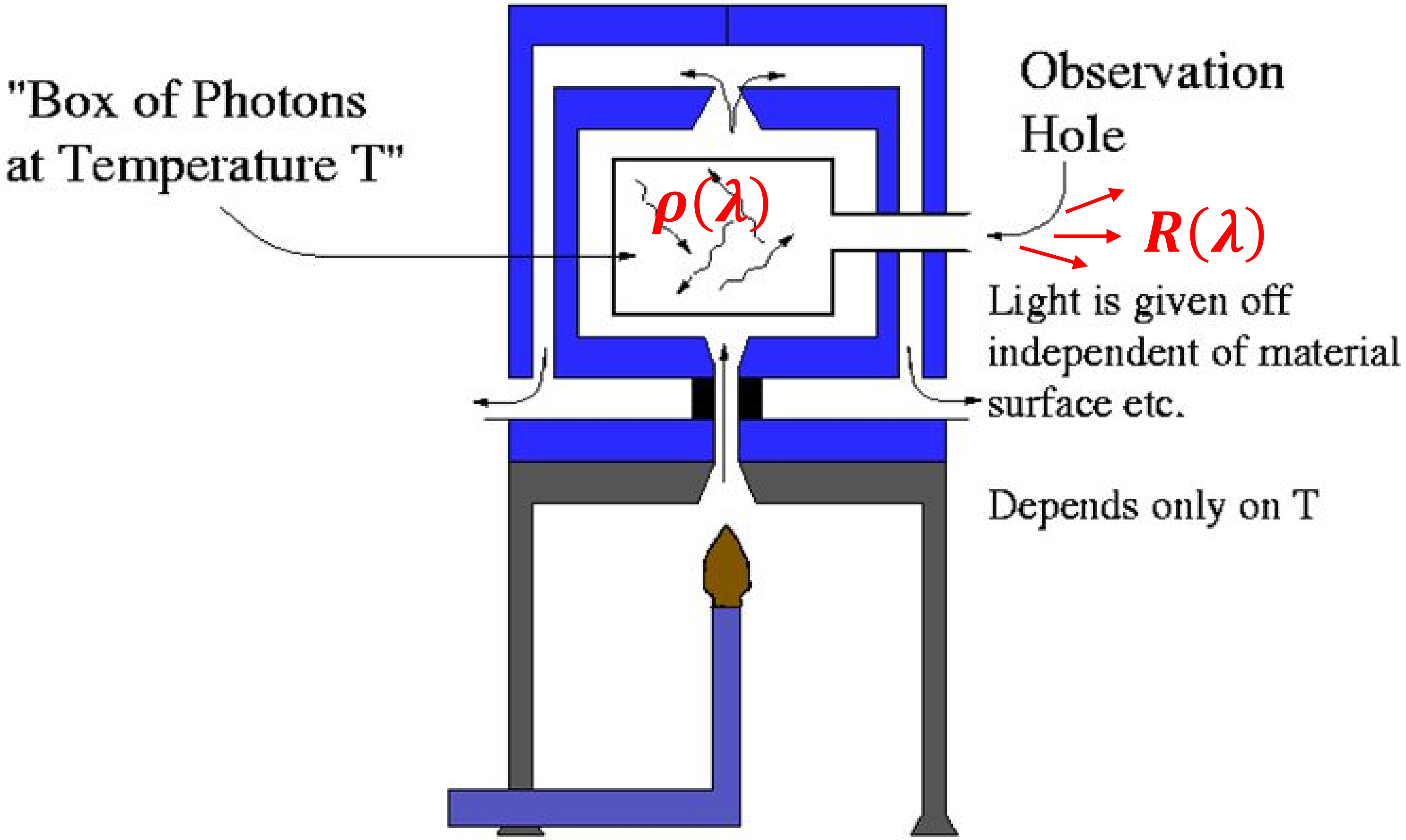
Wien Displacement Law

$$\lambda_{max} \sim 1/T$$

$$\lambda_{max}\, T = 2.897771955 \times 10^{-3}\ \text{m-K}$$



Blackbody Radiators

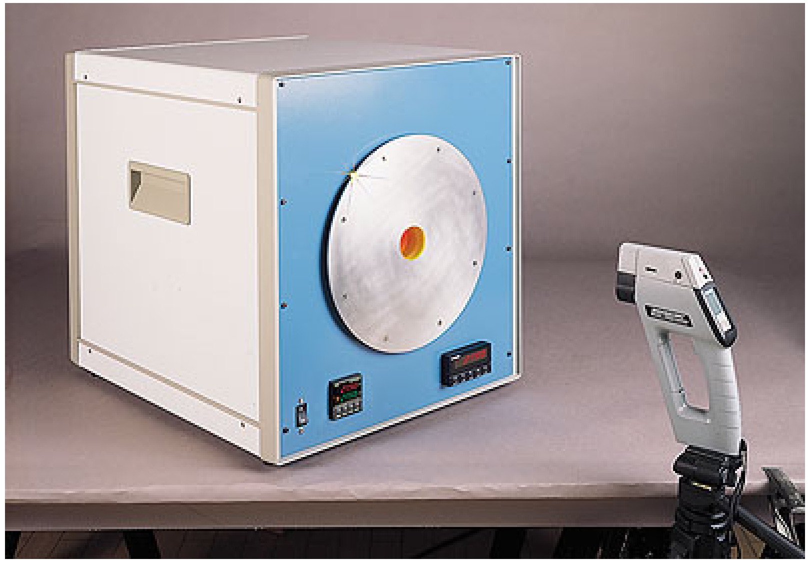


$$R(\lambda) = \frac{c}{4}\rho(\lambda)$$

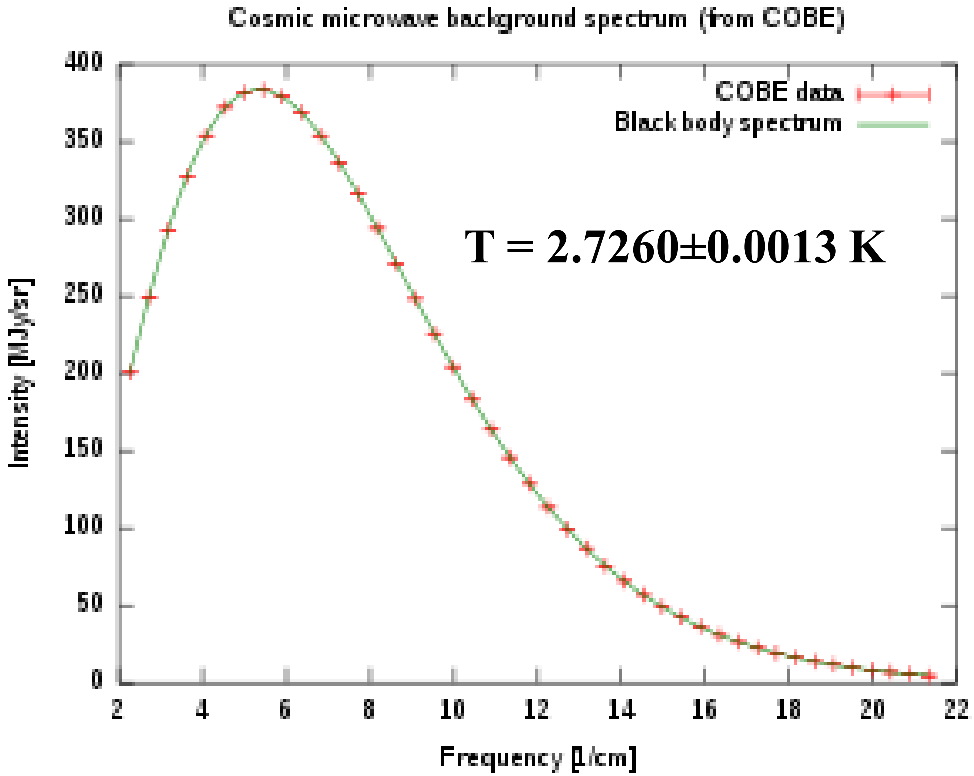
$$[\rho(\lambda)] = \frac{\text{Energy}}{\text{Volume}\cdot\text{Wavelength}} = \frac{\text{J}}{m^3\cdot\text{nm}}$$

$$[R(\lambda)] = \frac{\text{Power}}{\text{Area}\cdot\text{Wavelength}} = \frac{\text{W}}{m^2\cdot\text{nm}}$$

Commercial Blackbody Radiation Sources

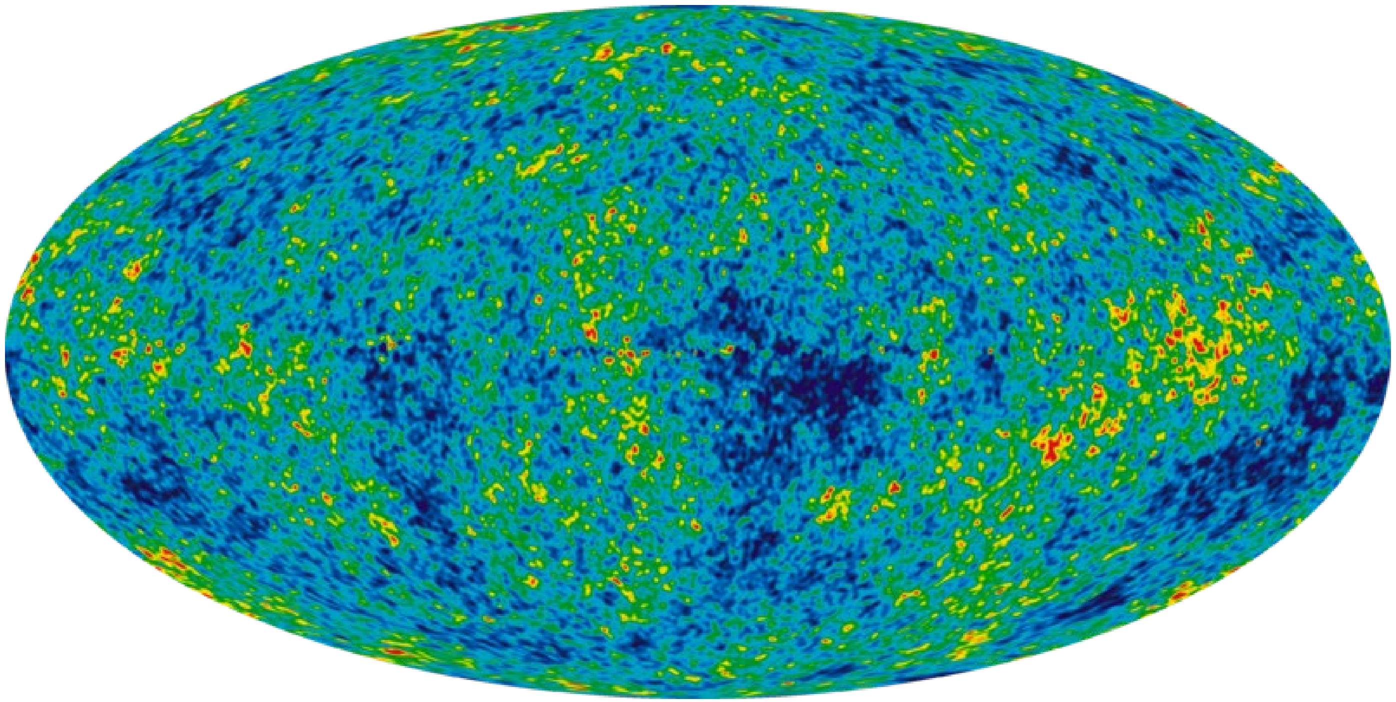


Cosmic Microwave Background Radiation Spectrum



T = 2.7260±0.0013 K

$$1\ cm^{-1} \approx 30\ GHz$$



The radiation is isotropic to roughly one part in 100,000: the root mean square variations are only 18 μK

https://en.wikipedia.org/wiki/Cosmic_microwave_background

What is the Energy Density of Electromagnetic Modes in the Box, $\rho(\lambda)$?

First do the classical calculation

Assume <u>Equipartition of Energy</u>

Every mode acquires $k_B T$ of energy, on average, for a system in thermal equilibrium

$$\boldsymbol{\rho(\lambda) = k_B T\, n(\lambda)}$$

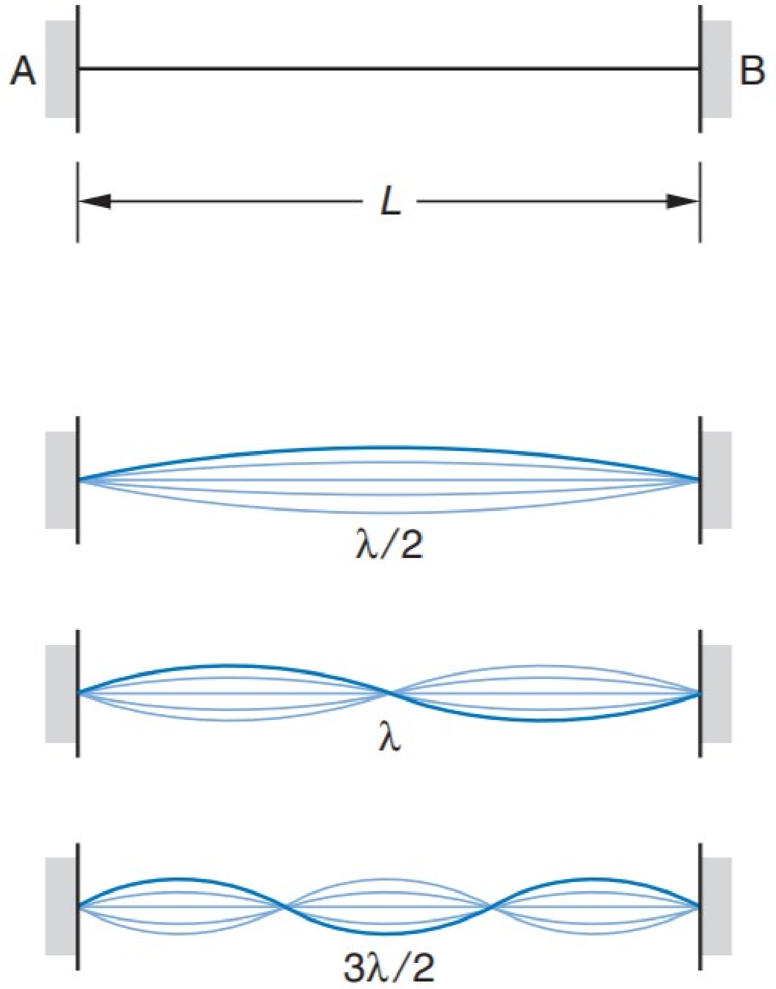
Energy per mode

Number of modes per unit volume at wavelength λ

What is the mode density of electromagnetic fields in the box of wavelength λ, $n(\lambda)$?

What is the Mode Density $n(\lambda)$?

Imagine all the resonant oscillation modes of a string of length L



+ many more modes!

The wavelengths of standing waves on the string are given by
$L = N\left(\frac{\lambda}{2}\right)$**, where** N **is an integer,** $N = 1, 2, 3, 4, \ldots$

We can ask the question: For a given wavelength λ**, what mode number does it correspond to? The answer is** $N = 2L/\lambda$**.**

Doing this in 3 dimensions for a cube of size $L \times L \times L$ **gives:**
$N = 4\pi L^3/3\lambda^3$.

The number of modes with wavelengths between λ **and** $\lambda + d\lambda$ **is given by:**

$$dN = \frac{4\pi L^3}{\lambda^4} d\lambda.$$

The number of modes per unit volume per unit wavelength is:

$$n(\lambda) = \frac{8\pi}{\lambda^4}$$ **(assuming 2 polarizations for each wavelength)**

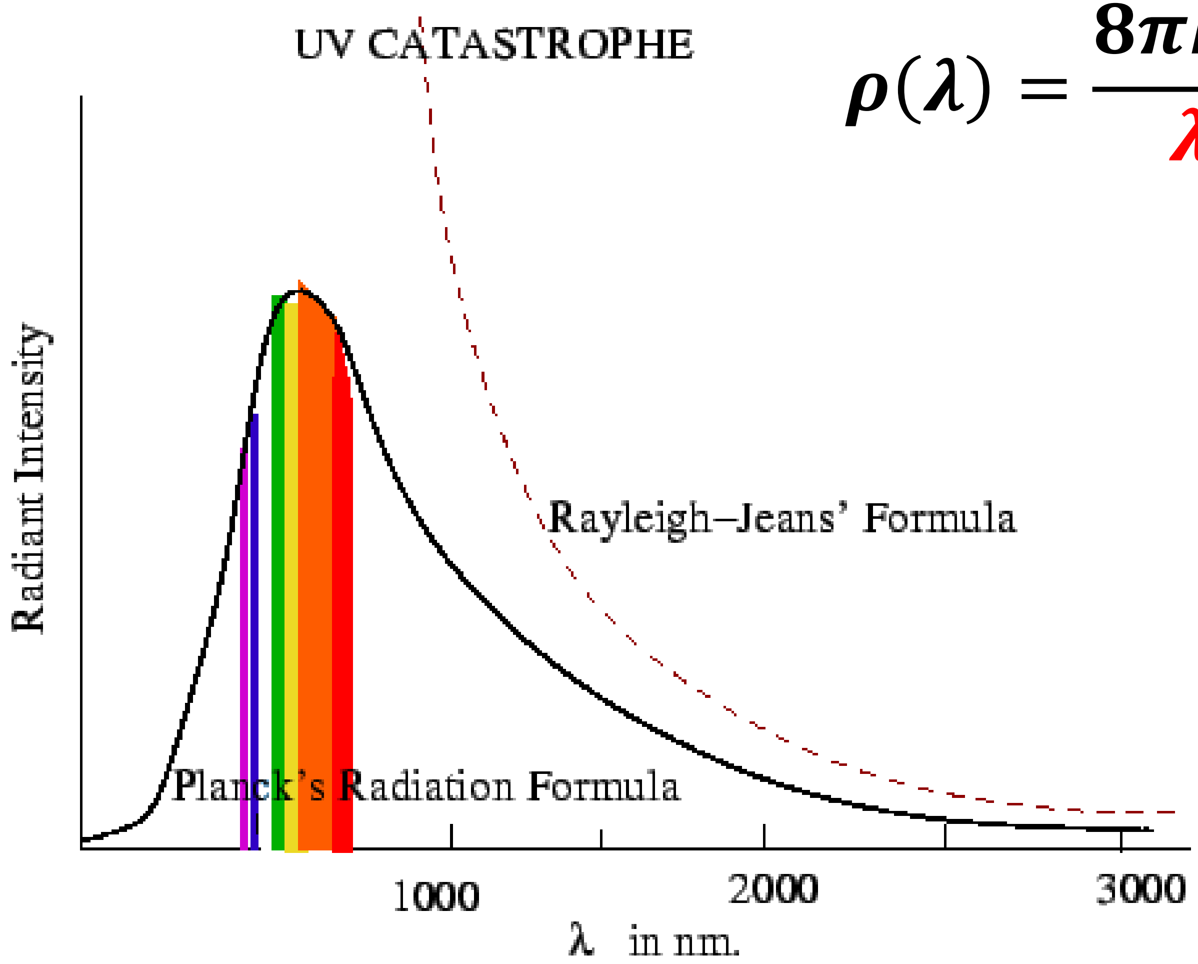
What is the Energy Density of Electromagnetic Modes in the Box, ρ(λ)?

Still doing the classical calculation

$$\rho(\lambda) = k_B T\, n(\lambda)$$

$$\rho(\lambda) = \frac{8\pi k_B T}{\lambda^4}$$

Rayleigh-Jeans Law

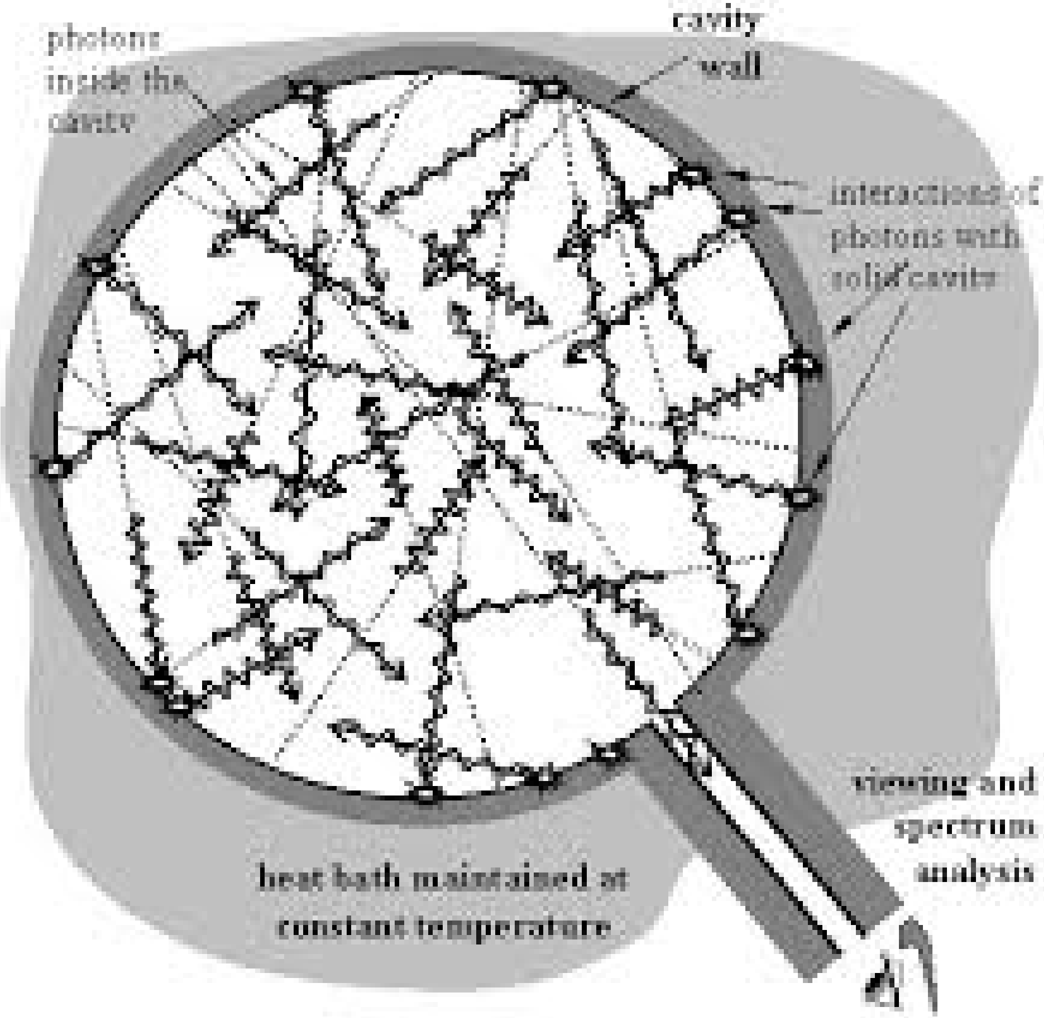


The ultra-violet catastrophe ensues…

Somehow we have to limit the occupation of the short-wavelength modes …

What is the Energy Density of Electromagnetic Modes in the Box, $\rho(\lambda)$?

The birth of Quantum Mechanics



The atoms in the walls of the box have electrons, and when these electrons vibrate at frequency f they emit electromagnetic waves at frequency f.

Likewise light at frequency f can be absorbed by the atoms in the walls and start oscillating at frequency f.

In equilibrium there is an equal energy flux from the walls to radiation and from radiation back into the walls.

What is the Energy Density of Electromagnetic Modes in the Box, $\rho(\lambda)$?

The birth of Quantum Mechanics

Max Planck (1900) made two (largely unjustified and revolutionary) assumptions:

1) **The atoms in the walls of the box have discrete energy levels given by $E_n = n\varepsilon$, where $n = 0, 1, 2, \ldots$ Hence the atoms can only interact with light of energy ε, 2ε, 3ε, etc.**

2) **The energy of light is directly related to the frequency of oscillation of the EM fields as $\varepsilon = hf$, where h is a *fudge factor* with units of J/Hz or $J \cdot s$. Surprisingly, this energy is independent of the intensity of the light.**

Why did Planck take $\varepsilon = hf$ and not some other dependence?

How Does This Get Rid of the Ultraviolet Catastrophe?

The likelihood of finding an atom in the walls of the box being excited to energy state E is given by:

$$g(E) = Ae^{-E/k_BT} \quad \text{(the Boltzmann factor from classical statistical mechanics)}$$

$g(E)$ is the fraction of atoms in the wall of the box that are excited to a state of energy E
T is the temperature of the walls of the box (K)
A is a normalization constant

This shows that the highly energetic atom energy states ($E \gg k_BT$) will be occupied with near-zero probability
Hence in equilibrium there will be very few highly energetic electromagnetic modes excited in the cavity

In other words, equipartition of energy breaks down for the energetic modes of the box

For example:

$$k_BT \sim 0.1\ eV$$

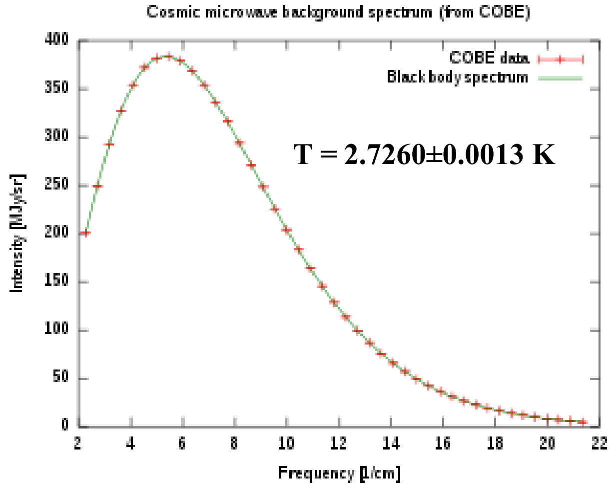
$$E \sim 2\ eV$$

$$e^{-E/k_BT} \sim 10^{-9}$$

$$\rho(\lambda) = \frac{8\pi hc/\lambda^5}{e^{hc/\lambda k_B T} - 1}$$

Planck Blackbody Radiation Formula

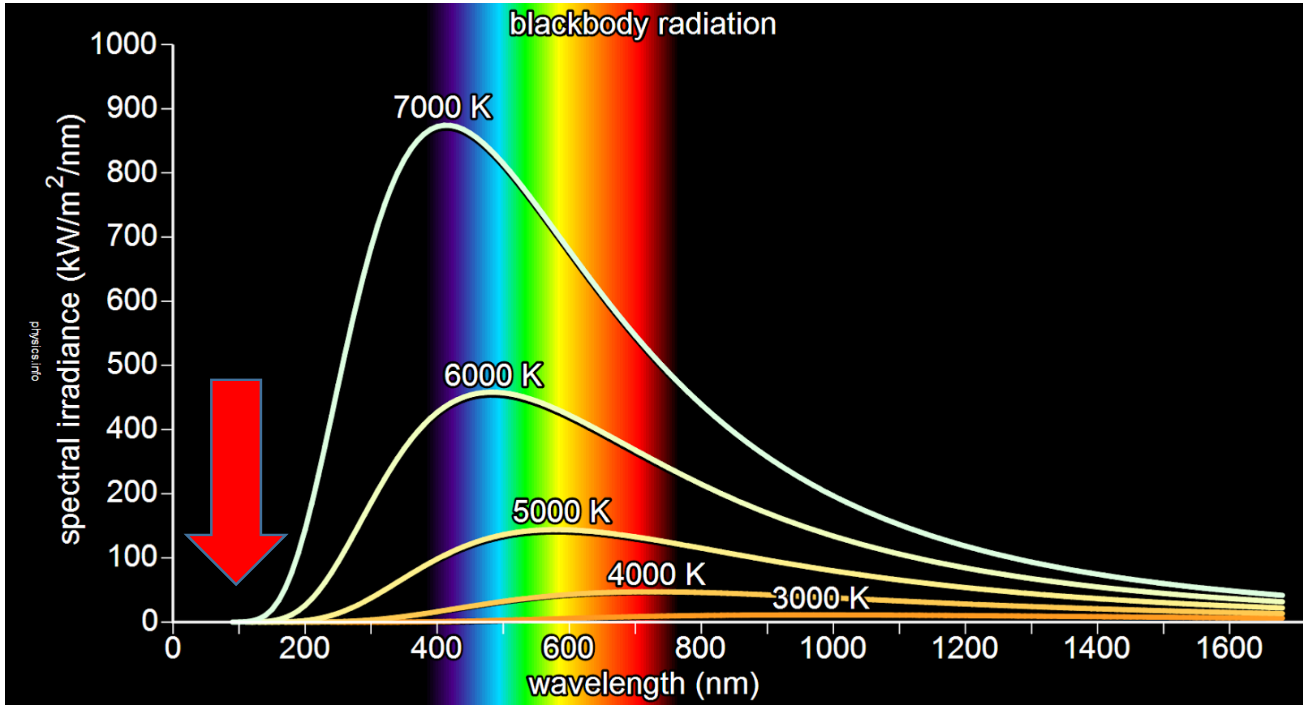
This equation fit the data very well!



The prediction remains finite as $\lambda \to 0$

$$\rho(\lambda) \approx \frac{8\pi hc}{\lambda^5} e^{-hc/\lambda k_B T}$$

This predicted exponential suppression at short wavelength is a characteristic property of blackbody radiation data



Planck has overcome the Ultraviolet Catastrophe, but at what price?

The value of Planck's constant

$$\boldsymbol{h = 6.626\ \times 10^{-34}\, J - s}$$
$$\boldsymbol{= 4.136\ \times 10^{-15}\, eV - s}$$

The reduced Planck constant: $\hbar = \boldsymbol{h/2\pi}$

Planck was obsessed with justifying these two assumptions in the 10 years or so that followed

He was a "reluctant revolutionary"

He was relieved when a young Albert Einstein picked up these ideas about light and showed that they could be used to understand the Photo-electric Effect (a totally different phenomenon), and determine the value of Planck's constant with high precision!